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Hawking Absorption and Planck Absolute Entropy of Kerr-Newman Black Hole

M. Hossain Ali[1,3] and Akhtara Banu[2]

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We find the existence of a quantum thermal effect, "Hawking absorption." near the inner horizon of the Kerr–Newman black hole. Redefining the entropy, temperature, angular velocity, and electric potential of the black hole, we give a new formulation of the Bekenstein–Smarr formula. The redefined entropy vanishes for absolute zero temperature of the black hole and hence it is interpreted as the Planck absolute entropy of the KN black hole.

KEY WORDS: inner horizon; outer horizon; Hawking absorption; entropy; Nernst theorem; Bekenstein–Smarr formula.

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1. INTRODUCTION

The discovery of Hawking (Hawking, 1974, 1975) that a black hole emits particles as a black body is one of the most important achievements of quantum field theory in curved spacetimes. In fact, due to Hawking evaporation classical general relativity, statistical physics, and quantum field theory are connected in quantum black hole physics. It is therefore, generally believed that the deep investigation of black hole physics would be helpful to set up a satisfactory quantum theory of gravity.

The area theorem and the Bekenstein–Smarr formula have established a remarkable relationship between thermodynamics and black hole physics. The area of a black hole event horizon (outer horizon) is regarded as the entropy (Hawking, 1972; Bekenstein, 1973, 1974) and the surface gravity of the horizon as the temperature (Bardeen *et al.*, 1973) of the black hole. The four laws of black

[1] Department of Applied Mathematics, Rajshahi University, Rajshahi – 6205, Bangladesh.
[2] Department of Mathematics, Naogaon Govt. College, Naogaon, Bangladesh.
[3] To whom correspondence should be addressed at Department of Applied Mathematics, Rajshahi University, Rajshahi – 6205, Banglades; e-mail: m_h_ali@bttb.net.bd.

hole physics (Bardeen *et al.*, 1973; Bekenstein, 1972; Smarr, 1973) were given and supported by Hawking radiation. However, there is still an open problem on black hole entropy (Unruh and Wald, 1982; Frolov and Page, 1993; Wald, 1994). According to the Nernst formulation of the third law of ordinary thermodynamics (often referred to as the Nernst theorem), the entropy of a system must approach zero as its temperature goes to zero. This assertion is commonly believed to be a fundamental law of thermodynamics. But the entropy of a black hole does not vanish as its temperature goes to absolute zero (Lee *et al.*, 1996; Wald, 1997).

In a recent work Zheng (1999) studied the thermal character of the inner horizon of the Kerr black hole and gave a new formulation of the Bekenstein–Smarr formula, which contains parameters of both the inner and outer horizons, and hence redefined the entropy of the black hole. The redefined entropy satisfies the Nernst theorem.

In this paper, applying Zheng's arguments we find "Hawking absorption" in the inner horizon, which gives a satisfactory explanation of the mechanism of the Hawking radiation of the Kerr–Newman black hole. We redefine the entropy of the KN black hole to satisfy the Nernst theorem and define its conjugate intensive variable, i.e., the redefined temperature. We also redefine the angular velocity and the electric potential of the black hole. We then reformulate the Bekenstein–Smarr differential formula in terms of these redefined variables, which was not derived in (Zheng, 1999). Unlike the Kerr black hole (Zheng, 1999) the redefined temperature for the KN black hole does not vanish as the black hole temperature goes to absolute zero.

The Kerr–Newman black hole has the metric

$$\mathrm{d}s^2 = \Sigma\Delta^{-1}(\mathrm{d}r^2 + \Delta \mathrm{d}\theta^2) + \Sigma^{-1}\big(\sin^2\theta\ W_1^2 - \Delta W_2^2\big), \tag{1}$$

where

$$\Sigma(r,\theta) \equiv r^2 + a^2\cos^2\theta, \quad \Delta(r) \equiv r^2 - 2Mr + a^2 + q^2,$$
$$W_1 = a\mathrm{d}t - (r^2 + a^2)\mathrm{d}\varphi, \quad W_2 = \mathrm{d}t - a\sin^2\theta\ \mathrm{d}\varphi,$$

$q^2 = q_m^2 + q_e^2$, and M, a, q_e, and q_m are the mass, the angular momentum per unit mass, the electric charge, and the magnetic charge of the black hole, respectively. The electromagnetic vector potential associated with the metric (1) is

$$A_\mu\, \mathrm{d}x^\mu = \Sigma^{-1}(q_m \cos\theta\ W_1 + q_e\, r W_2). \tag{2}$$

The KN black hole has two coordinate singularities corresponding to the outer and inner horizons at

$$r_\pm = M \pm \sqrt{M^2 - a^2 - q^2}, \tag{3}$$

where $M^2 \geq a^2 + q^2$, and the equality holds in the extremal case, $r_+ = r_-$. The event horizon is defined by $r_H = r_+$. The KN metric (1) has a stationary limit surface at $r_{\text{reg}} = M + \sqrt{M^2 - a^2\cos^2\theta - q^2}$ called the ergosphere in which a particle cannot remain at rest as viewed from infinity. The angular velocities of the horizons are defined by

$$\Omega_\pm = \lim_{r\to r_\pm} \frac{-g_{t\varphi}}{g_{\varphi\varphi}} = \frac{a}{r_\pm^2 + a^2}, \tag{4}$$

while the electric potentials by

$$V_\pm = \frac{q_e r_\pm + a q_m}{r_\pm^2 + a^2}. \tag{5}$$

The surface gravities of the horizons are defined as (Zheng, 1981; Zheng *et al.*, 1981)

$$\kappa_\pm = \lim_{r\to r_\pm} \left(\pm b\sqrt{-\hat{g}_{tt}}\right) = \mp\frac{1}{2}\lim_{r\to r_\pm} \sqrt{\frac{-g^{rr}}{g^{tt}}} \cdot \frac{\partial_r(g^{tt})}{g^{tt}}, \tag{6}$$

where $b = \sqrt{g_{rr}}\, \mathrm{d}^2 r/\mathrm{d}s^2$ is the proper acceleration of a particle which rests outside the horizon and near it, and $\hat{g}_{tt} = g_{tt} - g_{t\varphi}/g_{\varphi\varphi}$ is the red shift factor. For the KN metric (1), we obtain

$$\kappa_\pm = \frac{r_+ - r_-}{2(r_\pm^2 + a^2)}. \tag{7}$$

Recently, Ho *et al.* (1997) calculated the entropy of the KN black hole via the brick-wall method in the form

$$S = \frac{4\zeta(4)}{(2\pi)^3} \frac{1}{\varepsilon} \ln\left(\frac{\delta}{2}\right) A_H, \tag{8}$$

where $\zeta(n)$ is the Riemann Zeta function and A_H the area of the event horizon of the hole. If the angular cut-off δ and the invariant distance cut-off ε satisfy the relation

$$\left(\frac{\delta}{2}\right)^\varepsilon = \exp\left(\frac{\pi}{45}\right), \tag{9}$$

the entropy of the scalar field becomes equal to the Bekenstein–Hawking entropy: $S = A_H/4$. At absolute zero temperature of the hole, i.e., in the extremal case: $a^2 + q^2 = M^2$ and $r_H = M$, this reduces to

$$S = \pi(M^2 + a^2), \tag{10}$$

which is nonvanishing, and hence the KN black hole violates the black hole analogy of the "Nernst theorem."

This paper is arranged as follows. In Section 2 we investigate the thermal character of the inner horizon of the KN black hole and find the existence of a quantum effect, "Hawking absorption"at the inner horizon. In Section 3 we write the Bekenstein–Smarr formula in a new form and derived the expression for the redefined entropy of the black hole. We then reformulate the Bekenstein–Smarr formula in terms of the redefined entropy and find the conjugate intensive variable of the redefined entropy as well as the redefined angular velocity and the redefined electric potential of the KN black hole. The redefined entropy satisfies the Nernst theorem and hence it can be regarded as the Planck absolute entropy of the KN black hole. Finally, in Section 4 we present our remarks.

2. HAWKING ABSORPTION

In this section we find the inner horizon of the KN black hole as a thermodynamic system. There exists a quantum process called "Hawking absorption," which explains the mechanism of the Hawking radiation of the hole.

The outer horizon of the KN black hole is a future horizon for the observer outside the hole ($r > r_+$), but the inner horizon is a "past horizon"for the observer inside the hole ($r < r_-$). That means, the inner horizon is a horizon of a white hole for the observer in the region $r < r_-$. Since the physical process near the white hole is a time reversal of the physical process near the black hole, we can expect "Hawking absorption" for white holes as one expects Hawking radiation for black holes. Hence, there is a "Hawking absorption" to the inner horizon of the KN black hole for observers inside the hole ($r < r_-$).

In the KN spacetime, the radical equation of the Klein–Gordon equation can be put in the form

$$\Delta \frac{\mathrm{d}^2\Phi}{\mathrm{d}r^2} + 2(r - M)\frac{\mathrm{d}\Phi}{\mathrm{d}r} = \left(\lambda^2 + \mu^2 r^2 - \frac{K^2}{\Delta}\right)\Phi, \tag{11}$$

with $K = (r^2 + a^2)\omega - ma + e(q_e r + aq_m)$. Here Φ, μ, ω, and e are, respectively, wave function, mass, angular momentum, and electric charge of Klein-Gordon particles, and λ is a constant from separating variables.

Near the horizon, the KG Eq. (11) reduces to

$$\frac{\mathrm{d}^2\Phi}{\mathrm{d}\hat{r}^2} + (\omega - \omega_\circ)^2\Phi = 0, \tag{12}$$

where $\omega_\circ = m\Omega_\pm - eV_\pm$ and $\hat{r}$ is the tortoise coordinate defined by

$$\begin{aligned} \frac{\mathrm{d}\hat{r}}{\mathrm{d}r} &= \pm\frac{r^2 + a^2}{\Delta}, \\ \hat{r} &= \pm\left[r + \frac{1}{r_+ - r_-}\left\{(2Mr_+ - q^2)\ln\frac{|\, r - r_+ \,|}{r_+}\right.\right. \\ &\qquad \left.\left. -(2Mr_- - q^2)\ln\frac{|\, r - r_- \,|}{r_-}\right\}\right], \end{aligned} \tag{13}$$

where "+" is for $r > r_+$ and "−" for $r < r_-$. By studying the KG equation near the outer horizon, one can find the existence of Hawking radiation emitted from the horizon. We here are interested only in the case near the inner horizon. The solutions of (12) are

$$\Phi = e^{\pm \mathrm{i}(\omega - \omega_\circ)\hat{r}}. \tag{14}$$

Introducing the retarded Eddington–Finkelstein coordinate $u = t - [(\omega - \omega_\circ)/\omega]\hat{r}$, the outgoing and ingoing waves are, respectively, given by

$$\Phi_{\text{out}} = \exp[-\mathrm{i}\omega t + \mathrm{i}(\omega - \omega_\circ)\hat{r}] = e^{-\mathrm{i}\omega u}, \tag{15}$$

$$\Phi_{\text{in}} = \exp[-\mathrm{i}\omega t - \mathrm{i}(\omega - \omega_\circ)\hat{r}] = e^{-\mathrm{i}\omega u} e^{-2\mathrm{i}(\omega - \omega_\circ)\hat{r}}. \tag{16}$$

Since $r \to r_-$ corresponds to $\hat{r} \to -\infty$, and $\hat{r} \to 0$ as $r \to 0$, Eq. (15) is just the outgoing wave emitted by the inner horizon. However, Eq. (16) represents the ingoing wave to the inner horizon. Obviously,

$$\hat{r} \sim \frac{1}{2\kappa_-} \ln(r_- - r) \tag{17}$$

as $r \to r_-$. Hence,

$$\Phi_{\text{in}} = e^{-\mathrm{i}\omega u}(r_- - r)^{-\mathrm{i}(\omega - \omega_\circ)/\kappa_-}. \tag{18}$$

Using $| r_- - r | e^{\mathrm{i}\pi} = (r - r_-)e^{\mathrm{i}\pi}$, Φ_{in} can be analytically extended around the inner horizon r_- along the upper semicircle of radius $| r - r_- |$ in the complex r-plane, into the "one-way membrane" region ($r_- < r < r_+$). Then, Φ_{in} in the region $r_- < r < r_+$ is

$$\Phi_{\text{in}} = \Phi'_{\text{in}} e^{\pi(\omega - \omega_\circ)/\kappa_-}, \tag{19}$$

where

$$\Phi'_{\text{in}} = e^{-\mathrm{i}\omega u}(r - r_-)^{-\mathrm{i}(\omega - \omega_\circ)/\kappa_-} = e^{-\mathrm{i}\omega u} e^{-2\mathrm{i}(\omega - \omega_\circ)\hat{r}}. \tag{20}$$

The total wave function is given by

$$\Phi_\omega = N_\omega \left[Y(r_- - r)\, \Phi_{\text{in}}(r_- - r) + e^{\pi(\omega - \omega_\circ)/\kappa_-} Y(r - r_-)\, \Phi'_{\text{in}}(r - r_-) \right], \tag{21}$$

where

$$Y(r) = \begin{cases} 1, & r \geq 0, \\ 0, & r < 0. \end{cases} \tag{22}$$

We obtain

$$(\Phi_\omega, \Phi_\omega) = N_\omega^2 \left\{ 1 \pm \exp\left(\frac{\omega - \omega_\circ}{K_B T_-} \right) \right\} = \pm 1. \tag{23}$$

This indicates that the inner horizon absorbs thermal radiation from the region $r < r_-$. The thermal spectrum and temperature of this radiation are, respectively, given by

$$N_\omega^2 = \left[\exp\left(\frac{\omega - \omega_\circ}{K_B T_-}\right) \pm 1\right]^{-1}, \tag{24}$$

$$T_- = \frac{\kappa_-}{2\pi K_B}. \tag{25}$$

Hence, there exists some radiation from the region $r < r_-$ to the inner horizon, which is thermal radiation with temperature T_-. This thermal radiation is absorbed by the inner horizon and the corresponding quantum effect is named "Hawking absorption."

The KN black hole is stationary, so its outer horizon is in thermal equilibrium with the thermal radiation outside the black hole. The inner horizon is also in thermal equilibrium with the thermal radiation in the region $r < r_-$. Thus, the inner horizon not only absorbs thermal radiation at temperature T_-, but also emits thermal radiation at the same temperature T_-. Hence, the inner horizon of the KN black hole can be regarded as a thermodynamic system with temperature T_-.

Thus the Hawking radiation of the KN black hole originates from thermal radiation near the singularity. The thermal radiation from the singularity is absorbed by the inner horizon. Traveling in the reversed time direction, it then transits the "one-way membrane" region at $r_- < r < r_+$ and arrives at the outer horizon. Being scattered by the outer horizon, it travels forward in time to infinity as Hawking radiation.

The areas of the horizons, $A_\pm$, are given by

$$A_\pm = \pm \int \sqrt{\gamma}\, \mathrm{d}\theta \mathrm{d}\varphi = \pm 4\pi \left(r_\pm^2 + a^2\right), \tag{26}$$

where $\gamma = (r_\pm^2 + a^2 + l^2)^2 \sin^2\theta$ is the determinant of the 2-dimensional metric on the outer/inner horizon. Since the inner horizon is like the horizon of a white hole, A_- is defined as minus. In the proceeding section we shall see that A_- gives a contribution, as A_+ does, to the entropy of the black hole.

3. PLANCK ABSOLUTE ENTROPY

In this section, we first obtain the new formulation of Bekenstein–Smarr formula as was derived in (Zheng, 1999), and find the expression for the redefined entropy of the KN black hole. We then reformulate the Bekenstein–Smarr differential formula in terms of the redefined entropy and its conjugate intensive variable, i.e., the redefined temperature, and redefine the angular velocity, and the electric potential of the black hole. The redefined entropy is interpreted as the Planck absolute entropy of the black hole.

From (4), (7), and (26), we obtain

$$2\left(1+\frac{q^2}{2a^2}\right)\Omega_\pm J \pm q^2\kappa_\pm = r_\mp = M \mp \sqrt{M^2-a^2-q^2}, \tag{27}$$

$$\frac{1}{4\pi}\kappa_\pm A_\pm = \pm\sqrt{M^2-a^2-q^2}. \tag{28}$$

Adding (28) to (27), we get the Bekenstein–Smarr integral formulae as follows:

$$M = \left(\frac{1}{4\pi}A_+ + q^2\right)\kappa_+ + 2\left(1+\frac{q^2}{2a^2}\right)\Omega_+ J, \tag{29}$$

$$M = \left(\frac{1}{4\pi}A_- - q^2\right)\kappa_- + 2\left(1+\frac{q^2}{2a^2}\right)\Omega_- J. \tag{30}$$

Combining Eqs. (29) and (30), we get

$$\begin{aligned} M = {} & \left(\frac{1}{8\pi}A_+ + \frac{q^2}{2}\right)\kappa_+ + \left(1+\frac{q^2}{2a^2}\right)\Omega_+ J \\ & + \left(\frac{1}{8\pi}A_- - \frac{q^2}{2}\right)\kappa_- + \left(1+\frac{q^2}{2a^2}\right)\Omega_- J. \end{aligned} \tag{31}$$

This is a new one of the Bekenstein–Smarr integral formula as was derived in (Zheng, 1999). It contains parameters of both the inner and outer horizons. Eqs. (29), (30), and (31) are equivalent to each other.

From (27) and (28), we obtain

$$\begin{aligned} 2J\delta\Omega_\pm = {} & \mp\frac{2a^2}{2a^2+q^2}\frac{2(a^2+q^2)r_\pm - q^2M}{(2Mr_\pm - q^2)\sqrt{M^2-a^2-q^2}}\,\delta M \mp \frac{2a^2q^2}{2a^2+q^2}\,\delta\kappa_\pm \\ & \pm\frac{2a}{2a^2+q^2}\frac{a^2(2M^2-q^2)\pm q^2M\sqrt{M^2-a^2-q^2}}{(2Mr_\pm - q^2)\sqrt{M^2-a^2-q^2}}\,\delta a \\ & \pm\frac{2a^2q}{(2Mr_\pm - q^2)\sqrt{M^2-a^2-q^2}}\delta q, \end{aligned} \tag{32}$$

$$\begin{aligned} \frac{1}{4\pi}A_\pm\delta\kappa_\pm = {} & \pm\left[\frac{(2a^2+q^2)r_\pm}{(2Mr_\pm - q^2)\sqrt{M^2-a^2-q^2}} \mp 1\right]\delta M \\ & \mp\frac{(2M^2-q^2)a}{(2Mr_\pm - q^2)\sqrt{M^2-a^2-q^2}}\,\delta a \end{aligned}$$

$$\mp \frac{(2a^2 + q^2)q}{(2Mr_\pm - q^2)\sqrt{M^2 - a^2 - q^2}} \delta q. \tag{33}$$

Differentiating Eqs. (29) and (30), and using (32) and (33), we have

$$\delta M = \frac{1}{8\pi} C_+ \kappa_+ \delta A_+ + C_+ \Omega_+ \delta J + 2C_+ \kappa_+ q \delta q, \tag{34}$$

$$\delta M = \frac{1}{8\pi} C_- \kappa_- \delta A_- + C_- \Omega_- \delta J, \tag{35}$$

where

$$C_\pm = \left(1 \mp \frac{2\pi q^2 r_\pm \Omega_\pm}{a A_\pm \kappa_\pm}\right)^{-1}. \tag{36}$$

Equations (34) and (35) are just the differential forms of (29) and (30). Then the new BS differential formula corresponding to Eq. (31) takes the form

$$\begin{aligned} \delta M = {} & C_+ \left(\frac{1}{16\pi} \kappa_+ \delta A_+ + \Omega_+ \delta J_+ + \kappa_+ q_+ \delta q_+\right) \\ & + C_- \left(\frac{1}{16\pi} \kappa_- \delta A_- + \Omega_- \delta J_-\right), \end{aligned} \tag{37}$$

where

$$J_+ = J_- = J/2, \quad q_+ = q. \tag{38}$$

Equations (34), (35), and (37) are mathematically equivalent. We write (37) as follows:

$$\delta M = T_+ \delta S_+ + C_+ \Omega_+ \delta J_+ + C_+ \kappa_+ q_+ \delta q_+ + T_- \delta S_- + C_- \Omega_- \delta J_-, \tag{39}$$

where

$$T_\pm = \frac{\kappa_\pm}{2\pi K_B}, \qquad S_\pm = \frac{K_B}{8} \int C_\pm \mathrm{d}A_\pm \tag{40}$$

are, respectively, the temperature and the entropy of the outer/inner horizon.

In physical content, the new BS differential formula (39) is different from the usual one (34). Equation (34) states that the KN black hole is a single thermodynamic system composed of the outer horizon only, where as (39) shows it as a complex thermodynamic system composed of two subsystems, the outer horizon and the inner horizon. The temperature of the outer horizon is just the usual black hole temperature given by κ_+ in (7) and that of the inner horizon is the same as that given by κ_- in (25).

Equations (39) and (40) state that the entropy of the black hole receives contributions from both the outer and inner horizons. Thus, the entropy of the black hole is redefined and the "redefined entropy" is

$$\tilde{S} = S_+ + S_- = \frac{K_B}{8}\left(\int C_+ \mathrm{d}A_+ + \int C_- \mathrm{d}A_-\right). \tag{41}$$

The new entropy $\tilde{S}$ is different from the usual entropy S given in (8), which contains only the area of the outer horizon. In terms of the redefined entropy $\tilde{S}$ the BS differential formula is obtained as follows:

$$\delta M = \tilde{T}\delta\tilde{S} + \tilde{\Omega}\delta\tilde{J} + \tilde{V}\delta\tilde{q}, \tag{42}$$

where

$$\begin{aligned}
\tilde{T} &= \frac{1}{4\pi K_B}\frac{n_1}{d}, \quad \tilde{\Omega} = \frac{n_2}{d}, \quad \tilde{V} = \frac{n_3}{d}, \quad \tilde{J} = J, \quad \tilde{q} = q,\\
d &= 2(2M^2 - q^2)(4M^2a^2 + q^4)\sqrt{M^2 - a^2 - q^2}\\
&\quad + Mq^2\left\{4(a^2 + q^2)(q^2 - M^2) - q^4\right\},\\
n_1 &= 4\sqrt{M^2 - a^2 - q^2}\left\{(4M^2a^2 + q^4)\sqrt{M^2 - a^2 - q^2} - Mq^2(2a^2 + q^2)\right\}\\
&\quad + q^4(a^2 + q^2),\\
n_2 &= a\left\{2(4M^2a^2 + q^4)\sqrt{M^2 - a^2 - q^2} - Mq^2(2Ma^2 + q^2)\right\},\\
n_3 &= q\left\{2M(4M^2a^2 + q^4)\sqrt{M^2 - a^2 - q^2} + q^4(a^2 + q^2)\right\},
\end{aligned} \tag{43}$$

with $\tilde{T}$ the redefined temperature, $\tilde{\Omega}$ the redefined angular velocity, and $\tilde{V}$ the redefined electric potential of the black hole.

The redefined temperature $\tilde{T}$ does not vanish but the temperatures of the outer and inner horizons $T_\pm$ go to absolute zero as the black hole approaches the extremal limit: $M^2 \to (a^2 + q^2)$, $r_+ = r_- = M$. It is interesting that unlike the entropy S in (8), the redefined entropy $\tilde{S}$ in (41) approaches zero (as $C_\pm \to 0$) in this case. The absolute zero temperature corresponds to the extremal black hole. Thus the redefined black hole entropy $\tilde{S}$ satisfies the Nernst theorem. Hence, it can be regarded as the Planck absolute entropy of the black hole.

4. CONCLUDING REMARKS

The main concern of this study is to investigate the thermal character of the inner horizon, redefine the entropy to satisfy the Nernst theorem, and reformulate the Bekenstein–Smarr formula in terms of the redefined entropy and thereby find out the expressions for the redefined temperature, angular velocity, and electric potential of the Kerr–Newman black hole.

We detect a quantum process called "Hawking absorption" at the inner horizon, which explains nicely the mechanism of Hawking radiation of the KN black hole. The Killing vector $(\partial/\partial t)^{\alpha}$ is spacelike in the "one-way membrane" region located for the KN black hole between the outer and inner horizons, $r_- < r < r_+$, where negative-energy particles and antiparticles may exist. The particle- antiparticle pairs created by vacuum fluctuation outside and near the horizon may be realized by the tunnel-effect (Hawking, 1975). The region inside the inner horizon ($r < r_-$) contains no "one-way membrane," so negative-energy particles and antiparticles cannot travel forward in time transiting the region ($r < r_-$) to the singularity. The negative-energy antiparticle traveling in the reversed time direction from the inner horizon to the singularity is equivalent to the positive-energy particle traveling forward in time from the singularity to the inner horizon. At the inner horizon, the positive-energy particle arriving forward in time from the singularity cancels out the negative-energy antiparticle arriving in the reversed time direction from the outer horizon and transits the "one-way membrane" region. Thus the inner horizon produces radiation from the singularity. The radiation is absorbed by the inner horizon. This quantum effect is referred to as "Hawking absorption." Thus there is a flow of positive-energy particles produced near the singularity, which propagate forward in time and reach at the inner horizon. Being scattered by the inner horizon and transiting the "one-way membrane" region these particles go in the reversed time toward the outer horizon. At the outer horizon, they are scattered again and then travel forward in time to infinity as Hawking radiation.

We find the KN black hole as a complex thermodynamic system composed of two subsystems, the outer horizon and the inner horizon. For this complex system we have rewritten both the Bekenstein–Smarr integral formula and the differential formula as was done in (Zheng, 1999). The new formula are mathematically equivalent to the old formula, but are different in physical content. The new formula give the same temperature for the outer horizon as the usual one. The temperature of the inner horizon is the same as that of "Hawking absorption." The entropy of the black hole is redefined. The redefined entropy is different from the well-known one. The usual entropy contains the area of the outer horizon only, while the new entropy contains contributions from the areas of both the inner and outer horizons. In terms of this redefined entropy, we further rewrite the Bekenstein–Smarr differential formula and redefine the temperature, the angular velocity, and the electric potential of the KN black hole. It is interesting that the redefined entropy approaches zero as the black hole temperature goes to absolute zero. Thus the new entropy satisfies the Nernst theorem and hence, it can be interpreted as the Planck absolute entropy of the KN black hole. We also note that unlike the Kerr black hole (Zheng, 1999), the redefined temperature does not vanish for the absolute zero temperature of the black hole.

This work is interesting in that we not only have obtained the Planck absolute entropy but also have expressed the Bekenstein–Smarr formula in terms of this entropy and its conjugate intensive variable, and redefined the angular velocity and the electric potential of the hole. The results reduce to those of the Kerr black hole (Zheng, 1999) for $q = 0$, while the choice $a = 0$ gives the results for the Reissner-Nördstrom black hole, an interesting hole which permits its interpretation as a soliton (Hajicek 1981; Gibbons, 1982) and admits supersymmetry in the context of $N = 2$ supergravity (Gibbons, 1982; Aichelberg and Güven, 1981, 1983; Das and Freedman, 1977, 1976) in the extremal limit.

The redefined entropy, the reformulation of Bekenstein–Smarr formula, and the explanation of the Hawking radiation by the quantum process, Hawking absorption, for the black hole are interesting from the physical point of view. Hence the work is well justified.

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